5.1 Lecture Notes
Math 206
04/03/08
Mammenga

AREAS AND INTEGRALS

A. Definition: Let f be a function defined for $a \leq x \leq b$. Either of the equivalent expressions

$$\int_a^b f \qquad \text{or} \qquad \int_a^b f(x)dx$$

denotes the signed area bounded by $x = a$, $x = b$, $y = f(x)$, and the x-axis.

B. Rules:
(i) $\int_a^b (f \pm g) = \int_a^b f \pm \int_a^b g$
(ii) $\int_a^b (kf) = k \int_a^b f$
(iii) If $f \leq g$ for every x in $[a, b]$, then $\int_a^b f \leq \int_a^b g$
(iv) $\int_a^b f = \int_a^c f + \int_c^b f$ for any c in $[a, b]$
(v) $\int_a^b f = -\int_b^a f$

C. Definition: Let f be defined on $[a, b]$. The *average value* of f is given by $\dfrac{\int_a^b f(x)dx}{b-a}$.

TEAM ACTIVITIES:

1. How much area is "under" the graphs of the following curves between $[-3, 3]$?
(a) $y = 9$
(b) $y = 9 - x$
(c) $y = 9 - x^2$

2. Evaluate the following integrals.
(a) $\int_2^5 \frac{1}{4} dx$
(b) $\int_{-1}^1 \left(t^{71} + 5t^{27} - t^{13}\right) dt$
(c) $\int_{-2}^1 3u du$
(d) $\int_{-5}^0 |x + 3| dx$

3. Use the facts that $\int_0^3 f = 4$, $\int_3^5 f = -1$, and $\int_5^{10} f = 2$ to evaluate the following.
   (a) $\int_0^5 f$
   (b) $\int_3^{10} f$
   (c) $\int_0^{10} f$
   (d) $\int_0^3 \pi f$
   (e) $\int_0^{10} -2f$
   (f) $\int_{10}^5 (f+1)$

ASSIGNMENT:

1. Consider the equation $x^2 + y^2 = 9$.
   (a) Sketch the graph of the equation.
   (b) What is the area of region contained inside the graph?
   (c) What is the *signed* area of the region contained inside the graph?

2. Let $f(x) = \begin{cases} \frac{3}{5}x, & 0 \le x \le 5; \\ 3, & 5 < x \le 8; \\ -\frac{3}{2}x + 15, & 8 < x \le 10. \end{cases}$
   (a) Sketch the graph of $f(x)$.
   (b) Compute $\int_0^8 f(x)dx$.
   (c) Compute $\int_8^{10} f(x)dx$.
   (d) Compute $\int_5^0 f(x)dx$.
   (e) Compute $\int_0^{10} 7f(x)dx$.

3. Let $g(x) = \frac{2}{3}x - 2$.
   (a) Compute $\int_0^3 g(x)dx$.
   (b) Compute $\int_0^{12} g(x)dx$.
   (c) Compute $\int_0^{12} (g(x)+1)\,dx$.
   (d) Compute $\int_9^{-3} g(x)$.