4.5 Lecture Notes
Math 206
03/07/08
Mammenga

RELATED RATES

A. The typical related rates problem involves several amounts that vary together, and the task is to compute a particular rate of change given information about a different rate of change. To solve these sorts of problems, try the following steps:
   1. Write an equation that relates two or more variables that both vary with respect to a specific variable (usually *time*).
   2. Differentiate both sides with respect to the specific variable, using the chain rule appropriately.
   3. Solve this new equation for the desired rate.

TEAM ACTIVITIES:

1. Let x and y be variables that both vary with respect to time. Suppose that x and y are related by the equation $3x^2 - xy - y^2 = 1$, and suppose that at the moment that $x = 1$ and $y = -2$, x is increasing at a rate of 2 units per second. How fast is y increasing or decreasing at this same moment?

2. A baseball diamond is a square 90 ft on a side. A runner travels from home plate to first base at 20 ft/sec. How fast is the runner's distance from second base changing when the runner is halfway to first base?

3. The length of the base of a right triangle is increasing at the rate of 2 inches per minute. At the same time, the height of the triangle is decreasing in such a way that the length of the hypotenuse remains 10 inches. When the length of the base is 6 inches,
   (a) how quickly is the height of the triangle changing?
   (b) is the area of the triangle increasing?

4. A coffee filter has the shape of an inverted cone. Water drains out of the filter at a rate of 10 cm^3/min. When the depth of the water in the cone is 8 cm, the depth is decreasing at 2 cm/min. What is the ratio of the height of the cone to its radius?

ASSIGNMENT:

1. The base of a right triangle is increasing at a rate of 3 cm/sec and its height is decreasing at a rate of 2 cm/sec. At what rate is the area increasing/decreasing when the base has a length of 10 cm and the height has a length of 14 cm?

2. A highway patrol plane flies 1 mile above a straight section of rural interstate highway at a steady ground speed of 150 miles per hour. The pilot sees an oncoming car and determines that the line-of-sight distance from the plane to the car is 1.2 miles and that this distance is decreasing at a rate of 115 miles per hour. What is the speed of the car?

3. You are filling a hemispherical pool with water from a hose at a constant rate. You notice that the depth of the water is increasing at 1 in/min when the depth of the water is 18 inches. If the radius of the pool is 5 feet, how fast is the water entering the pool from the hose (in cubic inches per second)? You will need to use the fact that the volume of the bottom portion of a sphere with radius r is given by

$$V = \pi D^2 \left(r - \frac{1}{3} D \right),$$

where D is the depth of the bottom portion of the sphere.