3.3 Lecture Notes
Math 206
02/21/08
Mammenga

IMPLICIT FUNCTIONS AND IMPLICIT DIFFERENTIATION

A. If an equation involves both x and y, it may be very difficult to solve the equation for y. In this case, we say that the equation is an *implicit function*. That is, y is defined implicitly as a function of x.

B. To differentiate an implicit function, we must differentiate both sides of the equation (treating y as a function x) and use the chain rule whenever differentiating y. Then solve for $\frac{dy}{dx}$, if possible.

TEAM ACTIVITIES:

1. Consider the equation $5x^2 - 6xy + 5y^2 = 16$.
   (a) Using Derive, graph the equation.
   (b) Suppose that (a, b) is a point on the graph. Show that $(-a, -b)$ must also be a point on the graph.
   (c) Find $\frac{dy}{dx}$.
   (d) Find the slope of the tangent line at (a, b) and $(-a, -b)$.
   (e) Find (exactly) where the line $y = -x$ intersects the ellipse.
   (f) Find $\frac{dy}{dx}$ at each of the points that you found in part (e).

2. Find $\frac{dy}{dx}$ for $y^2 - x^2 = x^2y^2$. Could we have found the derivative explicitly?

3. Find an equation of the line that is tangent to the curve
$$xy + x^{2/3} + y^{2/3} = 2$$
at the point $(1, 1)$.

ASSIGNMENT:

1. Consider $7x^2y - \pi y = x^3$.
   (a) Compute $\frac{dy}{dx}$ using implicit differentiation.
   (b) Solve the equation for y.
   (c) Differentiate y with respect to x explicitly.

2. Find $\frac{dy}{dx}$ for $\left(x + \frac{y}{x}\right)^2 = 4x + y$.

3. Find the equation of the line tangent to $\cos x + \sin y = xy$ at the point $\left(\frac{\pi}{6}, 2.61457\right)$. You will need to use Derive to do this, and please round the constants to the nearest thousandth.

4. Consider $y^2 - x^2 = 1$. Find two points where the tangent line to the curve has a slope of $-\frac{1}{2}$. You should be able to find these values *exactly* (that is, no decimals)!