1.7 Homework Solutions
Math 206
Mammenga

THE GEOMETRY OF HIGHER-ORDER DERIVATIVES

ASSIGNMENT:

1. Suppose that you have a function f whose graph is pictured in the corresponding jpg. Note that the stationary points have coordinates $\left(3 - \frac{7\sqrt{3}}{3}, 13.202\right)$ and $\left(3 + \frac{7\sqrt{3}}{3}, -13.202\right)$, and the point of inflection has coordinates $(3, 0)$.
   (a) For which values of x, if any, is $f''(x)$ positive?
   Solution: We know that $f''(x)$ is positive wherever f is concave up. According to the graph, that interval of x-values is $(3, \infty)$.
   (b) For which values of x, if any, is $f''(x)$ negative?
   Solution: Again, this would be the same collection of x-values where f is concave down. Using the graph, we see that is the interval $(-\infty, 3)$.
   (c) For which values of x, if any, is $f''(x)$ zero?
   Solution: The inflection point, $x = 3$, is where $f''(x) = 0$.

2. Suppose that f is any linear function. This means that f can be written as $f(x) = mx + b$ for some values of m and b. Find the value of $f''(x)$ for *any* value of x and explain your reasoning. *(Hint: it may be useful to first determine $f'(x)$)*

   Solution: Using the hint, we first investigate the value of $f'(x)$. Since f is a line with slope m, we know that the slope of f at *any* value of x is m. That means that $f'(x) = m$. If we graphed the derivative function $f'(x) = m$, we would have a horizontal line that has a constant y-value of m. Now, $f''(x)$ is just the slope of f' at x. Since f' is a horizontal line, it has slope equal to zero at *any* value of x. Hence, $f''(x) = 0$.

3. Use the graph of $h''(x)$, shown in the corresponding jpg, to answer the following questions about h and h'.
   (a) On which intervals is h concave up?
   Solution: We know that h is concave up wherever $h''(x)$ is positive. According to the graph, that is the pair of intervals $(-\infty, 2) \cup (5, \infty)$.
   (b) On which intervals is h' decreasing?
   Solution: Having h' decreasing is the same thing as having $h'' < 0$. Thus, the interval $(2, 5)$ where where h' is decreasing.
   (c) Where are the point(s) of inflection for h?
   Solution: A point of inflection for h occurs precisely where $h''(x)$ equals zero. Hence, the points of inflection for h occur at the x-values 1, 2, and 5.
   (d) Rank the four numbers $h'(2)$, $h'(3)$, $h'(4)$, and $h'(5)$ in increasing order.
   Solution: Since the graph of h'' is negative over the interval $(2, 5)$, we have that h' is decreasing over this interval. Hence, the numbers written in increasing order are $h'(5)$, $h'(4)$, $h'(3)$, and $h'(2)$.

4. The graphs of g, g', and g'' appear in the corresponding jpg. Which graph is which function? How can you tell? You must justify your answers to receive full credit!

   Solution: First, notice that the local maxima and minima for the dark blue graph occur exactly where the fuschia graph has an x-intercept. This seems to indicate that the fuschia graph is the derivative of the dark blue graph. Now, the dark blue graph is concave up on the interval $(-4, 0)$, and this is precisely where the red graph is positive. This is also true for the interval $(4, 6)$. We can verify many other characteristics to be absolutely sure that the dark blue graph is g, the fuschia graph is g', and the red graph is g''.