1.5 Lecture Notes
Math 206
01/21/08
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ESTIMATING DERIVATIVES: A closer look

A. In the previous section, we said that the derivative of a function f at a point x is the slope of the graph of f at x. Let's practice estimating the slope of a given function at a particular x-value. (What if f is a constant function? What if f is a linear function?)

TEAM ACTIVITIES:

1. Open Derive on your computer and plot the expression $\frac{1}{4}\left(x^4 - 6x^3 + 3x^2 + 10x\right)$ for $-2 \leq x \leq 6$ and $-12 \leq y \leq 4$. Our goal is to estimate the slope of the graph at $x = 1$.
   (a) What is the $y-$coordinate of the point on the graph with $x = 1$? Plot this point (we'll call it P).
   (b) Choose a point on the graph that is to the right of $x = 1$ (we'll call it P_1). Find the equation of the line that passes through P and P_1. Use Derive to graph this line on the same coordinate system.
   (c) Find the point on the graph that has its x-coordinate half-way between the x-coordinates of P and P_1 (call this point P_2). Again, find the equation of the line that passes through P and P_2 and graph it on Derive.
   (d) In a similar manner to that of part (c), find P_3 and graph the line that passes through P and P_3.
   (e) Similarly, find P_4 and graph the line that passes through P and P_4.
   (f) Fill in the following blanks:

   Slope of the line containing P and P_1 = ________________

   Slope of the line containing P and P_2 = ________________

   Slope of the line containing P and P_3 = ________________

   Slope of the line containing P and P_4 = ________________

   Estimation of the slope of the tangent line = ________________

ASSIGNMENT:

1. Consider the function $f(x) = \cos x$.
   (a) Compute $f(1.37)$, $f(1.38)$, $f(1.39)$, $f(1.40)$, $f(1.41)$, $f(1.42)$, and $f(1.43)$, rounding to 6 significant digits.
   (b) Using the values you found in part (a), give an estimation of $f'(1.4)$.
   (c) Find the equation of the line that passes through $(1.4, f(1.4))$ and has slope equal to the estimation you found in part (b). (That is, let $m = f'(1.4)$.)
   (d) Use Derive to graph both $f(x)$ and the line you found in part (c) on the same set of coordinate axes. Make a sketch of these two graphs.

2. Consider the function $g(x) = \left(\frac{11}{8}\right)^x$.
   (a) Sketch the graph of $g(x)$, using Derive for help if needed.
   (b) Estimate $g'(0)$, $g'(1)$, and $g'(2)$, and describe how you found these estimations.
   (c) Sketch the tangent lines to $g(x)$ at $x = 0$, $x = 1$, and $x = 2$.
   (d) What seems to be true about the slope of the tangent line to $g(x)$ at *any* value of x?