1.4 Homework Solutions
Math 206
Mammenga

AMOUNT FUNCTIONS AND RATE FUNCTIONS: The idea of the derivative

SOLUTIONS:

1. Suppose that an object is moving away from the origin in the negative $x$-direction and that its speed is steadily decreasing. Sketch the object's
   (a) position vs. time graph;
   Solution: Please see the corresponding jpg for a graph.
   (b) velocity vs. time graph;
   Solution: Please see the corresponding jpg for a graph.
   (c) acceleration vs. time graph.
   Solution: Please see the corresponding jpg for a graph.

2. Suppose that an object moves from the origin to the right, then back to the origin. Sketch the object's
   (a) position vs. time graph;
   Solution: Please see the corresponding jpg for a graph.
   (b) velocity vs. time graph;
   Solution: Please see the corresponding jpg for a graph.
   (c) acceleration vs. time graph.
   Solution: Please see the corresponding jpg for a graph.

3. Find an algebraic expression for $f(x)$ if $f(0) = 5$ and
   (a) $f'(x) = 0$;
   Solution: First, note that $f'(x) = 0$. This means that at any value of $x$, the slope of the tangent line to $f$ at $x$ is 0. So, the graph of $f$ has a *constant* slope, which means it must be a line! Now, to write the equation of a line, we need to know a point it passes through and its slope. Since $f(0) = 5$, that graph of $f$ passes through $(0, 5)$. Since $f'(x) = 0$, we know that the graph of $f$ has a constant slope of 0. Hence, using the point-slope form of a line, $y - y_1 = m(x - x_1)$, we have $y - 5 = 0(x - 0)$, which is equivalent to $y = 5$. Therefore, $f(x) = 5$.
   (b) $f'(x) = -2$.
   Solution: Since $f'(x) = -2$, we have that the graph of $f$ has a constant slope of $-2$. Hence, $f$ is a line and we can write $y - 5 = -2(x - 0)$, which is the same as $y = -2x + 5$. Therefore, $f(x) = -2x + 5$.

4. Suppose that a body was discovered outside of Walker Science Center on a day in December when the temperature was 20° F. Let $T(t)$ represent the temperature (in degrees Fahrenheit) of the body $t$ hours after the body expired. Assume that the body had a temperature of 98.6° F at the time of death.
   (a) What does $T(4)$ represent?
   Solution: Since $T(t)$ represents the temperature of the body $t$ hours after the body expired, we have that $T(4)$ represents the temperature of the body four hours after the person died.

(b) Is $T'(1) > 0$? Justify your answer.
Solution: We are interested in whether the derivative of $T(t)$ is positive when $t = 1$. That means, we want to know if the rate of change of the temperature of the body is positive (meaning the temperature is increasing) or negative (meaning the temperature is decreasing). Since the body had an initial temperature ($98.6^\circ$) that was greater than the temperature of its surroundings ($20^\circ$), the temperature should decrease until it becomes the same as its surroundings. Hence, $T'(1) < 0$, and we would have to answer "no".

(c) Does there exist a value $t$ that will make $T(t) = 0$? Explain your reasoning.
Solution: The question really amounts to the following: "Is there a time when the body's temperature will be $0^\circ$?" Since the temperature of the body's surroundings is $20^\circ$, the body's temperature will decrease until it hits this temperature, and then stay at that value. Hence, there does not exist a value of $t$ so that $T(t) = 0$.